MATHEMATICS AND INNATE KNOWLEDGE: FROM PHILOSOPHICAL POSITIONS OF ANCIENT GREEKS TO NEUROSCIENCE

1. Introduction

It is difficult to find a scientific field in which Hellenic thought did not have a decisive impact. For example, the work of Thucydides provides a monumental reference to political science that remains a necessary tool for understanding international relations to this day. The materialism of Democritus and of the Epicureans, as well as the position of Heraclitus that "everything is in a state of flux" («*Τὰ πάντα ῥεῖ*») constitute the basic elements of dialectic materialism. The assertion of Epictetus that "sadness is caused not by facts themselves, but by our opinion about them" was the motivation for the development of the theory of cognitive psychology. Is there a more important text on Democracy than the *Funeral Oration*, attributed by Thucydides to Pericles?

Ancient Greek thought was not created in vacuum. Porphyry [1] mentions that Pythagoras was taught by the Egyptians, the Chaldeans and the Phoenicians. The great respect of this Neoplatonic philosopher for the Egyptians becomes evident from his frequent correspondence with the priest Anevo (probably a pseudonym of Jamblichus), whom he asks in one of his letters: «Is the prime cause identical or prior to the Demiurge»? In his work *On the caves of the Nymphs in Odyssey*, he mentions that «Nymphs symbolize the sensible world into which the souls have descended and from which they try to escape»; thus, Homer becomes a precursor of Plato. Pherekydes, who bequeathed his library to Thales of Miletus, was considered wise because he had studied Phoenician books. Jamblichus [2], who was a student of Porphyry, in his book *Mysteries*, notes that the Platonic theory of "corrupted souls" is consistent with the Egyptian belief that apocalypses to mortals by gods slowly degenerate with time. In his book *About souls*, and in other of his books, Hierocles [3] points out that there is consistency between the theological positions of Pythagoras and Plato on the one hand, as these are presented in the *Unwritten Doctrines* and in the *Dialogues*, and the corresponding positions of the "ancients" (i.e. the Chaldaeans and the Egyptians) on the other hand, as these are presented in the *Golden Verses* and in the *Chaldaean Oracles*. However, he points out an important difference: the positions of Pythagoras and, later, of Plato, manifest, a scientific character. Syrianus [4], who belonged to the Athenian philosophical school established by Jamblichus, believed that there is a close connection between theology, philosophy and certain ideas of great poets, such as Homer and Orpheus, since all of them were inspired by gods. Proclus [5] notes that Greek philosophy was indeed inspired by Orphic mystagogy and the apocalypses of the "ancients". In particular, according to him, Pythagoras learned about the mysteries of gods from the "ancients" and Plato learned about gods from Pythagoras and Orpheus; however, Proclus believes that Plato was the first real scientist. In his opinion, even Pythagoras cannot be considered a scientist, because his philosophy had an apocalyptic character in contrast with the apodictical character of Plato's own philosophy.

In the first part of this article we shall concentrate on philosophical positions of Ancient Greeks regarding mathematics.

2. Mathematics and the realm of ideas

The core of the Platonic theory is the existence of the following realities: (a) the intelligible (νοητὸς κόσμος) and (b) the sensible (αἰσθητὸς κόσμος). In addition, a quasi intermediate reality is implicitely supposed to exist at the level of understanding (διάνοια). In each of them there exist corresponding forms. The intelligible, or ideal forms, are created by gods (*παρὰ θεοῖς*). The discursive forms, which exist in the quasi intermediate reality, are projections of the ideal forms and function as demiurgic principles.

Mathematical entities exist in the quasi intermediate, discursive reality. As mentioned in the *Republic*, mathematics leads people away from the perceptible reality and prepares them for their avocation with the highest of sciences, namely Dialectics. According to Plato, mathematics is incomplete, since it does not examine the principles (axioms) from which it derives. These principles are examined by Dialectics, the only perfect science. Mathematics functions as a bridge: on the one hand, it tends towards perfection, since it illustrates in a paradigmatic way characteristics of gods and of perfect forms; on the other hand, its incompleteness becomes evident from the fact that it refers to perceptible reality, such as the calculation of the orbits of stars. It should be noted that, according to Platonism, the more sensible something is, the less significant it turns out to be. For Plato, as well as for most of the Neoplatonic philosophers, mathematics helps us understand the realm of ideas «by analogy» [6]. As it is mentioned in *Timaeus* «everything is ruled by analogies». For example, Proclus writes: «Mathematics is perfectly suitable for revealing to corrupt souls truths about gods; Plato explained to us many wonderful doctrines about the gods by means of mathematical forms, whereas Pythagoras concealed his secret theological initiations by using mathematical veils». Jamblichus states that «if one wonders how the many could be in the One, and all in the Indivisible, let him think of the monad».

Let us repeat that, according to Plato, mathematics provides projections of ideas, which are the utmost realities. Therefore, by analyzing the properties of numbers, one can glimpse at the characteristics of gods. This is the explanation of the statement made by Pythagoras himself, according to which «everything is made of numbers» (*ἀριθμῷ δὲ τὰ πάντα πεποίηκεν*»). For instance, the mathematical notion of 'mean' leads to the notion of moral virtue as the mean of two extremes: excess and deficiency. So, the number 5, which is the mean of 1 and 9, symbolizes justice. Not only ethics but also physics borrow notions from mathematics. In *Timaeus*, it is claimed that the world's soul was shaped by the Creator according to a model of an ideal form. If the world were a surface, then only one mean would be sufficient, since there is only one mean between two square numbers (e.g. between 3^2=9 and 4^2=16 there is 3x4=12). But, since the world is three-dimensional, two means are required, and this is why there exist water and air between fire and earth (between two cubic numbers, there exist two means, e.g. between 2^3=8 and 3^3=27 there are 2^3x3=12 and 3^2x2=18).

Certain philosophers tried to elevate mathematics to an even higher level than that of Plato. Especially Nicomachus [6] (2nd century B.C.), the author of *Introduction to Arithmetics* as well as of *Theologoumena*, after examining every number from a physical, ethical and theological viewpoint, reaches the conclusion that numbers are not merely projections of perfect forms, but that their characteristics are precisely those of gods. This is why Photius [7] claims that «Nicomachus sought to transform numbers into Gods». In his ten famous books on *Pythagorism*, Jamblichus tried to 'Pythagorise', i.e. to mathematize the Platonic theory. According to him, since the sensible world is organized on the basis of numbers, mathematics not only subsumes physics and ethics, but also foreshadows Dialectics. Proclus tried to elevate geometry to a level similar to the one that Nicomachus had elevated arithmetic. For him, geometry, due to its images and syllogisms, is more suitable than arithmetic for teaching eternal truths to the fallen souls. Furthermore, Proclus believed that the reliance of mathematics on axioms should not be considered as a weakness, since axioms are innate in souls. According to him, the existence of axioms is consistent not only with the Metaphysics of Plato[1] but also with the position of Stoic philosophers that there exist universal truths, which are evident to everyone; furthermore, it is consistent with Aristotle's statement [8] that there exist truths, which cannot be proven, but which are intuitively evident. This qualification of axioms provides a firm scientific basis to mathematics: starting with innate truths and using Aristotelian logic, mathematics derives rigorous truths [9].

Euclid's *Elements* is perhaps the greatest scientific achievement of antiquity [10]. Proclus considered this work as an application of the Platonic philosophy to geometry. Indeed, in the same way that Dialectics is based on metaphysical axioms, Euclid's geometry is based on mathematical axioms. It is difficult to find another work that has had a stronger impact on the development of modern physics and mathematics than this monumental work. It should be emphasised that several modern thinkers, including Kepler, Descartes and Newton, studied Euclid's geometry in detail. Newton's differential calculus was formulated in a geometrical language precisely because this great scientist believed that a work is scientific only if it is written in the rigorous geometrical language of Euclid's *Elements*. Proclus played an important role in popularizing this work; in particular, his book *Comments on the first book of Euclid's Elements* led to *mathesis universalis* during the Middle Ages [11].

According to Platonism, essential mathematical entities exist independently of us in another non-material realm. Contrasting to this position, Aristotle believed that numbers and other mathematical notions may be suggested by perceptible objects through the process of abstraction. A fundamental disagreement in this respect persists even today. Indeed, modern "platonizing" thinkers like Russell, Gödel and Penrose [12], believe in the existence of a mathematical reality, independent of us; they, thus, claim that mathematics is discovered instead of being created. On the other hand, "anti-platonizing" thinkers do not believe in the existence of such a reality; for them, mathematics is a creation of humans and, specifically, of the human brain. Almost all biologists, but also some distinguished mathematicians, like E. Brian Davis, are "anti-platonizing" [13].

Galileo was the first to understand that the laws of nature are written in a mathematical language. Ancient Greeks did not use differential calculus, so they could not understand that particular laws of nature correspond to particular differential equations. However, they did come close to understanding this essential role of mathematics. For example, Jamblichus states, in a text preserved by Psellus [14] [15]: «Indeed, the same can be proven for the celestial rotations and for the formations made by stars as they move periodically. The shapes created and the forces which exist between them, as well as the il-

1. In this dialogue *Parmenides* he states that «there is only one beginning, there cannot be many» (137 d sq).

luminations of the moon and the order of the spheres and the distances between them and the centers of the circles on which they move, everything is expressed in numbers». In addition, Proclus wonders: «How is the sensible world organized? According to what principles? What principles was it born from, if not from mathematical ones?».

There is no doubt that our brain in general and our cognitive capacity in particular, are the result of Darwinian evolutionary processes. Cavemen did not invent the integers; they only invented names and symbols in order to express some concrete sums, such as the sum of their fingers. Later on, people starting with simple mathematical entities, like the integers, and using complicated brain processes were able to establish mathematical structures of great complexity. Therefore, Aristotle was right when claiming that numbers were created from sensible objects *via* abstraction; and "anti-platonizing" scholars are right as well, when ascertaining that mathematical structures are the creation of the human brain. However, in our opinion, this does not necessarily imply that there are no mathematical truths, which exist in a realm independent of us. Bellow, two arguments supporting the Platonic attitude are exposed:

A) There is experimental evidence that fundamental laws of nature correspond to particular mathematical equations. The famous equations of Schrödinger and of Einstein, for example, correspond to the laws which govern the infinitely small and the infinitely large, respectively. The laws of nature obviously have an objective hypostasis; therefore, by association, the equations, which correspond to them also acquire an objective hypostasis. The theories of quantum mechanics and of general relativity are inconsistent; thus, the great challenge for the theoretical physicists today is to discover the so-called "theory of everything", which will unify all physical interactions. If this mathematical formalism exists, then it already exists in the mathematical world of Plato.

B) We know since 1931 from the famous theorem by Gödel, that no mathematical logic is complete. This means that there is no system, in which, starting from a finite number of axioms we can answer whether every statement in this system is true or not. According to Alain Connes [16], Gödel's theorem does not express any weakness of mathematics, as it is usually stated, but on the contrary, it implies its objective hypostasis. Since, for example, true statements for positive integers cannot be proven by using only a finite number of axioms, it follows that the system of positive integers contains infinite information. But this is precisely one of the basic characteristics of objective reality: it cannot be qualified by using only a finite number of statements.

On the other hand, it is difficult to reject the "anti-platonizing" position, according to which the existence of mathematical equations, which correspond to laws of nature, is merely the consequence of the regularity of these laws, as well as of the ability of the brain to "mathematicize" [13].

In our opinion, the main problem with the Platonic attitude is the answer to the following question: Where is the Platonic mathematical realm located? For Plato, who accepted the existence of metaphysical worlds, this problem does not exist. But for someone who rejects any metaphysical conception, the answer to this question is of crucial importance. Perhaps this problem can be bypassed using the following definition:

Plato's mathematical world is by definition the abstract mathematical space (*Timaeus*, 52a) which consists of all possible true mathematical relations.

The existence of this space explains the common feeling many mathematicians experience at some precise monent [17] designated by the exclamation "Eureka", that is, the feeling that they have just discovered something that pre-existed. Furthermore, it provides a resolution to the conflict between "platonizing" and "anti-platonizing" attitudes.

We already mentioned the correspondence between the laws of nature and specific mathematical equations. But what exactly is meant by the term "correspondence"? Are the laws of nature expressed or determined by mathematical equations? In our opinion, the answer to this question is not so important; what is important is the role of mathematics in the process of understanding these laws, and we consider this role to be decisive. For example, the mathematical analysis of Schrödinger's equation provides the deepest possible understanding of the mysterious quantum world. But this equation, like other basic equations, is comprehended in some Platonic world (in the sense of the definition given earlier). Thus, it is only the visit to this world that allows us to understand the essence of the phenomena which correspond to mathematical equations. This reminds us of the position of the Platonic philosophers that deep understanding can be achieved only with respect to real forms (ideas) and not through their sensible (phenomenal) substitutes.

At this point, we should of course point out that «*Οὐδὲν ἀγαθὸν ἀμιγὲς κακοῦ*». Indeed, the apotheosis of the intelligible world, as opposed to the material world, had a negative impact on the development of experimental sciences.

3. Tabula rasa or tabula inscripta?

On the basis of Plato's dialogue *Phaedon* and its interpretation by the Neoplatonic philosophers[2], there exist the following categories of souls: at the first level, the souls of gods and their follow travelers and at the second level, the pure (*ἄχραντοι*) souls, which are sent to earth in order to save the third category of souls, namely those that are corrupt. Thus, Socrates was sent to earth on a soteriological mission, especially to save the souls of young people. This role of Socrates is similar to that of Pythagoras. Even the souls that have fallen from grace were pure some time earlier; they thus contain 'in themselves' an *a priori* knowledge. For this reason, the process of learning and discovering is merely a process of recollection. By stimulating these souls with appropriate questions, the teacher stimulates this process of recollection. Within this framework, the maieutic method of Socrates acquires an absolutely necessary character. After having lead with his questions, a slave to discover certain geometrical truths, Socrates exclaims: «The slave has always had this knowledge within his soul» (*Meno*, 86 b). The less corrupt a soul is, the less teaching it requires. Thus, according to Neoplatonic philosophers, Pythagoras needed only minor stimulation by the Barbarian wise men in order to discover himself the unwritten doctrines.

To this Platonic attitude on innate knowledge, which was further developed by Leibniz, Aristotle juxtaposed a mitigate one. Furthermore, Locke proposed that the human mind is a *tabula rasa*, i.e. a blank slate, which is filled only as a result of experience.

Who was right, Plato or Locke and his followers? It appears, they were both right. Indeed, recent studies show that basic geometrical notions [18], as well as our capacity of

2. In particular, cf. the *Commentary* by Syrianos, as reported by his student Hermias.

approximate arithmetic [19] are innate. Regarding the capacity for approximate arithmetic, we mention a publication in "Science" [19], where in one of the relevant experiments, children of the Munduruku indigene group were presented with 1 to 25 dots and were asked to find their number. The Munduruku, who live in an isolated area of the Amazon, have words only for numbers from 1 to 5. This experiment showed that the human brain has an innate capacity to calculate approximately, independently of language. Specifically, from 1 to 3 dots the answer was precise. But, for 3 to 25 dots the answer was approximate. For example, despite the fact that there is a word for 5, for 5 dots the answers were 3, 4, 5, and "about as many as the fingers of one hand". For 13 dots, among others, there was the answer "two hands and something more". Other similar experiments showed the ability of the children for addition and subtraction, but, again, approximately. It seems that for precise arithmetics (for numbers higher than 3), one requires the existence of a specific algorithm, as well as the existence of a specific word or at least of an abstract symbol for each number.

But what does the statement that there is an innate capacity for approximate arithmetic entail? Does an innate knowledge of mathematical notions exist? In our opinion, it does not. What does though exist in the brain is a biologically predetermined cognitive predisposition. The brain converts this predisposition into knowledge, by using intricate neuronal mechanisms which are crucially affected by the continuous bombardment of stimuli from the environment. Remarkable achievements in molecular biology, cellular biology and brain imaging techniques, have created for the first time in the history of mankind a framework for the study of the above brain processes. The neuroscientist O'Keefe [20] for example, by recording the electrical activation of brain cells in the hippocampus of mice, showed that the hippocampus creates a representation of the external space. This is accomplished by using certain cells which O'Keefe named "place cells". As the mouse moves in a cage, specific cells of its brain are activated only when the mouse is in a specific position. It seems that the brain subdivides the space in many overlapping areas and each one of them is represented in the brain by a specific cell. As Kandel emphasizes [21], the study of the brain shows that we need both Plato's *a priori* knowledge and Locke's (Aristotle's) experience. The general ability of the brain to create representations of space is innate, but a specific map is created only as a result of experience. The existence of neural circuits is an example of *a priori* knowledge. The behavior of place cells is the result of changes at the level of neural synapses and these changes are dictated by experience. It is remarkable that neuroscientists are now able to study these changes on a molecular level. In particular, Kandel has shown that the initial creation of a map requires a qualitative change at the synapses, but the maintenance of this map requires the creation of new synapses.

The study of a specific behavior requires the study of specific neural circuits. But how can we study *in vivo* these neuronal circuits? The new remarkable brain imaging techniques of Functional Magnetic Resonance Imaging (fMRI), of Proton Emission Tomography (PET) and of Single Photon Emission Computed Tomography (SPECT), allow us to locate the activation of neuronal circuits in specific areas of the brain. For example, using functional MRI, we can see that the areas activated during exact (but not approximate) arithmetic calculations, coincide with the areas of the brain relevant to language.

These imaging techniques, despite their major importance, cannot be used for the study of the dynamics of the brain, because they do not produce results in real time. For example, it is not possible using these techniques to answer the question whether the activation of certain areas in the brain during a specific mathematical calculation is due to the calculation itself or to the decision to perform this calculation. One of the most important techniques for studying the dynamics of the brain is the so called Magnetoencephalography (MEG). Communication in the brain is achieved by electrical signals. The magnetic field which is created by these signals can be measured outside the head, using an extremely sensitive apparatus. MEG is the technique of determining the electrical current from the knowledge of the magnetic field. There has been a significant improvement of this technique recently, which, in our opinion, will play an important role in the elucidation of consciousness. For example, it appears that complicated intellectual functions require synchronization of several neural circuits (the so-called binding problem) and this is achieved by stimulating neurons at frequencies between 40-80 Hertz. It is expected that this crucial aspect of consciousness can be studied using electromagnetoencephalography.

The above new developments may constitute elements of a general framework within which an answer will finally be given to the profound question posed by Plato: «How are human beings, whose contact with the world is so short, personal and restricted, able to know what they know?».

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